Alternative definition:- A function $f(x)$ is said to be continous at $x=a$ if

LHL = RHL = Value of the function

$$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

Def:- A function is said to be continous in open interval (a, b) if it is continous at all points of the open interval.

Def: A function is said to be continous in closed interval $[a, b]$ if it is

(i) continous in (a, b)

(ii) $\lim_{x\to a^+} f(x) = f(a+) = f(a)$

(iii) $\lim_{x\to b^-} f(x) = f(b^-) = f(b)$

Discontinuity:- If a function is not continous at $x=a$, then $x=a$ is called point of discontinuity. If the given function is not continous at every means discontinous at every point of the interval, then it ~~said~~ is called discontinous in that interval.

Kinds of discontinuity

(i) First Kind:- If left hand limit and right hand limit exist at a point $x=a$ but not equal, then it is called discontinuity of first kind.

$\Rightarrow$ LHL $\neq$ RHL at $x = a$

$\Rightarrow \lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

Ex. $f(x) = \frac{|x|}{x}$ has first kind of discontinuity of first kind at $x = 0$.

$\lim_{x \to 0^-} f(x) = f(0^-) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{|0-h|}{(0-h)}$

$= \lim_{h \to 0} \left(\frac{h}{-h}\right) = -1$

$\lim_{x \to 0^+} f(x) = f(0^+) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{|0+h|}{(0+h)} = \lim_{h \to 0} \left(\frac{h}{h}\right) = 1$

$\Rightarrow f(0^-) \neq f(0^+)$

$\Rightarrow$ First kind of discontinuity.

Removable discontinuity :- Means that the such type of discontinuity, which can be removed by redefining the function

Ex. $\lim f(a^-) = f(a^+)$ but they are not equal to $f(a)$

i.e $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$ & $f(1) = 1$

then define $f(x)$ s.t. $f(1) = 2$, then it is called removable discontinuity.

Similarly, we have infinite discontinuity

So $\lim_{x \to \pi/2^-} f(x) = -\infty$; $\lim_{x \to \pi/2^+} f(x) = \infty$

$\Rightarrow$ Both are Infinite limits.